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Monte Carlo approach to insurance ruin problems using conjugate processes *

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Abstract

In this paper is discussed a simulation method developed by S. Asmussen called conjugate processes which is based on a version of Wald's fundamental identity. With this method it is possible to simulate within finite time risk reserve processes with infinite time horizons. This allows us to construct Monte Carlo estimators for the ruin probability, which is one of the main problems in insurance risk theory. Some examples of the Poisson/Exponential and Poisson/Uniform cases are presented.

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1 Introduction

One of the main problems in insurance risk theory is to estimate the ruin probability [1-8,11]. It can be roughly described as follows.

The risk reserve process over $(0, t]$ is the difference between a premium deterministic process $u + ct$ and the accumulated claims Z_t (a compound Poisson process), for some given initial capital $u \geq 0$. The premium income rate c is fixed by the insurance company and is independent of t. The idea is to study the behavior of the risk reserve process that models the accumulated capital over finite or infinite time

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horizons, in particular the probability that exists a moment τ when the risk process is negative. This is called the ruin probability.

The main purpose of this paper is to introduce Monte Carlo estimators (MCEs) for the ruin probability in infinite time horizon using conjugate processes. This approach allows us to simulate within finite time a risk reserve process with infinite time horizon. Here, we construct a MCE for the ruin probability using the empirical distribution of the ruin events after a sufficiently large number of simulations. Some examples of the Poisson/Exponential (P/E) and Poisson/Uniform (P/U) cases are presented.

This paper is organized as follows. We begin in §2 by introducing basic terminology and notation. In §3 we show a formulation for the conjugate process and construct the MCEs. In §4 we compute examples of the P/E and P/U cases. Finally, §5 presents some concluding remarks.

2 Preliminaries

Assumption 2.1

(a) The claims arrive according to a Poisson process $\{N_t\}_{t\geq 0}$ with intensity λ and interclaim times $\{T_t\}_{t\geq 1}$.

(b) The claim sizes $X_1, X_2, \ldots$ are i.i.d nonnegative random variables with a finite mean μ.

(c) X_i and $\{N_t\}_{t\geq 0}$ are independent.

Definition 2.2 The accumulated claim process is $Z_t := \sum_{n=0}^{N_t} X_n$ for $t \geq 0$, with $X_0 := 0$.

We next recall the classical risk reserve process [2,7,8].

Definition 2.3 Let u be the initial capital and $c > 0$ be the premium income rate.

We define the risk reserve process

$$Y_t := Z_t - ct, \qquad t \in (0, \infty),$$

and the time to ruin

$$\tau := \inf \{t > 0 : Y_t > u\}.$$

Definition 2.4 A family $(F_\theta)_{\theta\in\Theta}$ of distributions on $\mathbb{R}$ is called a conjugate family if the F_θ are mutually equivalent with densities of the form

$$\frac{dF_\theta}{dF_{\theta_0}}(x) = \exp\{(\theta-\theta_0)x - h_{\theta_0}(\theta)\} \tag{1}$$

and if for some fixed $\theta_0 \in \Theta$ the parameter set Θ contains all $\theta \in \mathbb{R}$ for which (1) defines a probability density for some $h_{\theta_0}(\theta)$.

Then, by definition, $P_{\theta_0} := P$ is the probability law of the process Y_t. In addition, $\theta_0 < 0$ is the solution of

$$\phi'_X(-\theta_0) = c/\lambda,$$

where $\phi_X(\beta) := E\left(e^{\beta X}\right)$ is the moment generating function of X. This definition of θ_0 allows us to choose the sign of $E_\theta Y_t$ as we prove below (Proposition 2.7).

Also note that $\phi_{\theta_0}(\beta) = E_{\theta_0}\left(e^{\beta X}\right) = \phi_X(\beta)$.

Equation (1) implies that $h_{\theta_0}(\theta)$ is given in terms of the cumulant generating function of F_{θ_0} by

$$h_{\theta_0}(\theta) := \log E_{\theta_0} e^{(\theta-\theta_0)X}.$$

The accumulated claim process Z_t is a compound Poisson process, so its moment generating function [2] is

$$\phi_{Z_t}(\beta) = e^{\lambda t(\phi_X(\beta)-1)}. \tag{2}$$

Proposition 2.5 *Let $\theta, \theta_0 \in \Theta$ with $\theta \neq \theta_0$. Then*

$$\phi_\theta(\beta) = \frac{\phi_{\theta_0}(\beta+\theta-\theta_0)}{\phi_{\theta_0}(\theta-\theta_0)}.$$

Proof: Using (1)

$$
\begin{aligned}
\phi_\theta(\beta) &= \int_{-\infty}^{\infty} e^{\beta x} dF_\theta(x) \\
&= \int_{-\infty}^{\infty} e^{\beta x} \frac{e^{(\theta-\theta_0)x}}{E_{\theta_0} e^{(\theta-\theta_0)X}} dF_{\theta_0}(x) \\
&= \frac{\phi_{\theta_0}(\beta+\theta-\theta_0)}{\phi_{\theta_0}(\theta-\theta_0)}. \qquad \Box
\end{aligned}
$$

Proposition 2.6 $\frac{log E e^{\beta Y_t}}{t} = \lambda(\phi_X(\beta)-1) - \beta c.$

Proof: By definition of Y_t, we can see that

$$E e^{\beta Y_t} = \phi_{Z_t}(\beta)/e^{\beta c t}.$$

Hence, from (2) we have

$$E e^{\beta Y_t} = e^{\lambda t(\phi_X(\beta)-1)-\beta c t}.$$

Applying the log function to both sides of the latter equation and dividing by t, completes the proof. $\Box$

Now from Proposition 2.5

$$E_\theta e^{\beta Z_t} = E_{\theta_0} e^{(\beta+\theta-\theta_0)Z_t} / E_{\theta_0} e^{(\theta-\theta_0)Z_t}.$$

Using (2)

$$E_\theta e^{\beta Z_t} = e^{\lambda t \phi_{\theta_0}(\theta-\theta_0)(\phi_\theta(\beta)-1)},$$

which implies that under P_θ, Z_t is also a compound Poisson process with arrival rate $\lambda_\theta = \lambda\phi_{\theta_0}(\theta-\theta_0)$ and claims distribution F_θ. Therefore, replacing E with E_θ in Proposition 2.6 we obtain

$$\text{(3)} \quad \frac{\log E_\theta e^{\beta Y_t}}{t} = \lambda_\theta(\phi_\theta(\beta)-1) - \beta c = \lambda\phi_{\theta_0}(\theta-\theta_0)(\phi_0(\beta)-1) - \beta c.$$

Proposition 2.7 *If ϕ_X'' exists in an interval I that contains $-\theta_0$, then*

$$\mu_\theta := E_\theta Y_t > 0 \qquad \textit{when} \quad \theta > 0$$

and

$$\mu_\theta < 0 \qquad \textit{when} \quad \theta > 0.$$

Proof: Let $\chi_\theta(\beta) := \frac{\log E_\theta e^{\beta Y_t}}{t}$, so that from (3)

$$\chi_\theta(\beta) = \lambda_\theta(\phi_\theta(\beta) - 1) - \beta c.$$

In particular, if $\theta = \theta_0$,

$$\chi_{\theta_0}(\beta) = \lambda_{\theta_0}(\phi_{\theta_0}(\beta) - 1) - \beta c,$$

and taking $\beta = -\theta_0$ we have

$$\chi'_{\theta_0}(\theta_0) = \lambda\phi'_{\theta_0}(-\theta_0) - c.$$

On the other hand, recalling that $\phi_{\theta_0}(\beta) = \phi_X(\beta)$, we get

$$\phi'_X(-\theta_0) = c/\lambda = \phi'_{\theta_0}(-\theta_0),$$

which yields

$$\chi'_{\theta_0}(-\theta_0) = 0.$$

Also, for all $\beta \in I$

$$\chi''_{\theta_0}(\beta) = \lambda\phi''_X(\beta) = \lambda E_{\theta_0}(X^2 e^{\beta X}) > 0,$$

so that $-\theta_0$ is a local minimum, and $\chi_{\theta_0}(\cdot)$ is convex on I.

Now let $\theta \in \Theta$. Then Proposition 2.5 implies

$$\chi_\theta(\beta) = \chi_{\theta_0}(\beta + \theta - \theta_0) - \chi_{\theta_0}(\theta - \theta_0),$$

and, therefore,

$$\chi'_\theta(0) = \chi'_{\theta_0}(\theta - \theta_0), \tag{4}$$

and, moreover,

$$\chi'_\theta(\beta) = \lambda\phi'_X(\beta + \theta - \theta_0).$$

On the other hand,

$$\chi'_\theta(\beta) = (E_\theta Y_t e^{\beta Y_t})/E_\theta e^{\beta Y_t},$$

and so

$$\chi'_\theta(0) = E_\theta Y_t = \mu_\theta.$$

Using (4)

$$\mu_\theta = \chi'_{\theta_0}(\theta - \theta_0),$$

and

$$\mu_0 = \chi'_{\theta_0}(-\theta_0) = 0.$$

The last two equalities and the convexity of $\chi_{\theta_0}(\cdot)$ yield the desired conclusion. □

3 Monte Carlo estimators

Our main purpose in this section is to estimate the ruin probability considering an infinite time horizon for the risk reserve process. We first introduce some definitions.

Definition 3.1 Let u and τ be as in Definition 2.3. The ruin probability in finite time is

$$\Psi(u, T) := P(\tau < T),$$

and the ruin probability in infinite time is

$$\Psi(u) := P(\tau < \infty).$$

The premium income rate c is usually taken as

$$c = (1 + \rho)EZ_t/t.$$

As Z_t is a compound Poisson process, c is independent of t. The number ρ is called the *safety loading*, and is related to the capital expected growth as follows.

Proposition 3.2 *If $\theta > 0$, then $P_\theta(\tau < \infty) = 1$.*

Proof: Under the law P_θ, Z_t is a compound Poisson process with $N_t \sim \text{Poisson}(\lambda_\theta)$ and claim sizes $X_i \sim F_\theta$. Therefore

$$E_\theta(Z_t) = \lambda_\theta t E_\theta X.$$

Applying the strong law of large numbers to the accumulated claim process yields

$$\lim_{t\to\infty}\frac{1}{t}\left(Z_t-\lambda_\theta t E_\theta X\right)=0 \qquad \text{a.s.} \tag{5}$$

Now, from Proposition 2.7 we have $E_\theta(Z_t - ct) > 0$ and, therefore, $\rho < 0$, and using (5)

$$\lim_{t\to\infty}\frac{1}{t}\left(Z_t-\lambda_\theta t E_\theta X-\rho\lambda_\theta t E_\theta X\right)=-\rho\lambda_\theta E_\theta X>0 \qquad \text{a.s.}$$

This implies

$$(Z_t-\lambda_\theta t(1+\rho)E_\theta X)\to+\infty \qquad \text{a.s.,}$$

which completes the proof. □

Consider a conjugate family $(F_\theta)_{\theta\in\Theta}$ governing a random walk $\{S_t\}_{t\geq 0}$ in discrete or continuous time. Define $\mathfrak{F}_T := \sigma(S_t; t \leq T)$, with the usual extension to stopping times.

Next, we present the version of the Wald's fundamental identity used by Asmussen [1,2]. The proof can be seen in [3].

Theorem 3.3 *Let τ be a stopping time for $\{S_t\}_{t\geq 0}$ and $G\in\mathfrak{F}_\tau$, $G\subseteq \{\tau<\infty\}$. Then for each $\theta_0,\theta\in\Theta$*

$$P_{\theta_0}G=E_\theta\left[\exp\left\{(\theta_0-\theta)S_\tau-\tau\chi_\theta(\theta_0-\theta)\right\};G\right]. \tag{6}$$

From Definition 2.3 and (6)

$$\frac{dP_{\theta_0}}{dP_\theta}=\exp\left\{(\theta_0-\theta)Y_\tau-\tau\chi_\theta(\theta-\theta_0)\right\}, \tag{7}$$

and integrating (7) over $\{\tau<\infty\}$ we can express the ruin probability in infinite time as

$$\Psi(u)=E_\theta\left[\left(\exp\left\{(\theta_0-\theta)Y_\tau-\tau\chi_\theta(\theta-\theta_0)\right\}\right)\cdot I\left\{\tau<\infty\right\}\right].$$

Proposition 3.4 *Let $\theta > 0$. If we compute n simulations of the conjugate process*

$$R_\theta := \exp\left\{(\theta_0 - \theta)Y_\tau - \tau\chi_\theta(\theta - \theta_0)\right\},$$

then with probability 1

$$\frac{1}{n}\sum_{i=1}^{n} R_\theta^i \to \Psi(u) \quad \textit{as} \quad n \to \infty,$$

where R_θ^i is the final value of the realization of the conjugate process after simulation i $(i = 1, 2, \dots)$.

Proof: By Proposition 3.2 the ruin occurs almost surely, and so each of the n simulations of R_θ can be performed in a finite number of steps. Moreover, as

$$E_\theta R_\theta = \Psi(u),$$

by the strong law of large numbers it follows that, with probability 1,

$$\frac{1}{n}\sum_{i=1}^{n} R_\theta^i \to \Psi(u) \quad \text{as} \quad n \to \infty. \quad \square$$

We call $\frac{1}{n}\sum_{i=1}^{n} R_\theta^i$ a *Monte Carlo estimator* (MCE) for $\Psi(u)$.

Observe that integrating (7) over $\{\tau < T\}$ we can write the ruin probability in a finite time T as

$$\Psi(u, T) = E_\theta\left[\left(\exp\left\{(\theta_0 - \theta)Y_\tau - \tau\chi_\theta(\theta - \theta_0)\right\}\right) \cdot I\left\{\tau < T\right\}\right].$$

Then, in this case, the corresponding conjugate process is

$$R_\theta^T := \exp\left\{(\theta_0 - \theta)Y_\tau - \tau\chi_\theta(\theta - \theta_0)\right\} \cdot I\left\{\tau < T\right\},$$

and so we could construct an analogous MCE for $\Psi(u, T)$.

Remark 3.5 (a) Observe that if $\theta = \theta_0$, then $R_{\theta_0}^T = I\{\tau < T\}$. Thus to simulate $R_{\theta_0}^T$ is equivalent to simulate the original process Y_t, which, in insurance terminology is called a crude simulation [7,8].

(b) We can simplify R_θ^T taking θ as the Lundberg value $\theta_1 := \gamma + \theta_0$, where $\gamma > 0$ is the unique solution of Lundberg's equation $\chi_{\theta_0}(\gamma) = 0$. In this case, $R_{\theta_1}^T$ is called the Lundberg process. Using Proposition 2.7 one can see that $\chi_{\theta_1}(\theta_0 - \theta_1) = 0$, which implies that

$$R_{\theta_1}^T = \exp(-\gamma Y_\tau) \cdot I\{\tau < T\}.$$

Therefore, taking $\Delta > 0$, $\theta = (1+\Delta)\cdot\theta_1$ and using Theorem 3.3 we obtain the following expression:

$$R_{\theta_1(1+\Delta)}^T = \exp\{-(\gamma + \theta_1\Delta)Y_\tau + \tau\chi_{\theta_1}(\theta_1\Delta)\} \cdot I\{\tau < T\}. \tag{8}$$

(c) In (b), the corresponding variance $\sigma_\theta^2 = \mathrm{Var}_\theta R_\theta^T$ is

$$\sigma_{\theta_1(1+\Delta)}^2 = E_{\theta_1}\left[\exp\{-2(\gamma + \theta_1)Y_\tau + \tau\chi_{\theta_1}(\theta_1\Delta)\} \cdot I\{\tau < T\}\right] - \Psi^2(u, T),$$

and for the infinite horizon case is

$$\sigma_{\theta_1(1+\Delta)}^2 = E_{\theta_1}\exp\{-2(\gamma + \theta_1)Y_\tau + \tau\chi_{\theta_1}(\theta_1\Delta)\} - \Psi^2(u).$$

(d) The overshot $B(u)$ of the risk process, defined as $B(u) := Y_\tau - u$ is useful to calculate $\sigma_{\theta_1}^2$. It is known [1,2] that when the claims are exponentially distributed and the arrival process is Poisson (P/E case), $B(u)$ is exponentially distributed:

$$P_\theta(B(u) > b) = \exp(-b/E_\theta X). \tag{9}$$

4 P/E and P/U examples

4.1 Example P/E

The Poisson/Exponential case has been extensively researched [1-5] because it is easy to calculate the ruin probability for the infinite time horizon. It is a well known fact [5] that if the safety loading ρ is positive, then

$$\Psi(u) = \frac{1}{1+\rho} \exp\left(-\frac{\rho u}{\mu(1+\rho)}\right). \tag{10}$$

Let us consider the P/E case with $\mu := EX = 1$, $\lambda = 0.8$, $\rho = 0.1$ and $T = \infty$. The right-hand side of (10) depends on the initial capital u. Let $\Psi(u) = 0.05$. Then the initial capital is $u = 31.904$, and the premium income rate is $c = (1+\rho)\lambda\mu$ (remember that we deal with a compound Poisson Process Z_t).

We solve the Lundberg equation for γ using Proposition 2.6:

$$\chi_{\theta_0}(\gamma) = \lambda(\phi_X(\gamma) - 1) - c\gamma = 0.$$

Then $\gamma = 0$ is the trivial solution, and the other solution is

$$\gamma = (c-\lambda)/\lambda = 0.0909.$$

Now we calculate the variances:

$$\sigma^2_{\theta_0} = E_{\theta_0} I^2\{\tau < \infty\} - E^2_{\theta_0} I\{\tau < \infty\} = \Psi(u) - \Psi^2(u) = 0.0475$$

$$\sigma^2_{\theta_1} = \mathrm{Var}_{\theta_1} R_{\theta_1} = \mathrm{Var}_{\theta_1} e^{-\gamma T_\tau} = \mathrm{Var}_{\theta_1} e^{-\gamma(u+B(u))} = e^{-2\gamma} \mathrm{Var}_{\theta_1} e^{-\gamma B(u)}.$$

From (1), $E_{\theta_1} X = (1-\gamma)^{-1}$, which together with (9) implies that $B(u) \sim \exp(1-\gamma)$. Thus

$$E_{\theta_1} e^{-2\gamma B(u)} = (1-\gamma)/(1+\gamma) \qquad \text{and} \qquad E_{\theta_1} e^{-\gamma B(u)} = 1-\gamma.$$

Hence,

$$\sigma^2_{\theta_1} = e^{-2\gamma u}\left(\frac{1-\gamma}{1+\gamma} - (1-\gamma)^2\right) = 2.08 \times 10^{-5} < \sigma^2_{\theta_0}.$$

Note that the difference between the variances is significant, which is an statistical advantage [9] to construct confidence intervals for $\Psi(u)$.

To show some numerical results, let $\mu = 1$, $\lambda = 0.8$, $\rho = 0.1$, $c = 0.88$. ¿From Proposition 2.6, we can see that N^θ_t is a Poisson process with arrival rate c, and X_θ is exponentially distributed with parameter λ/c.

Moreover, $\gamma = 0.0909$, $\theta_0 = -0.0488$ and $\theta_1 = 0.0421$.

One can compare the theoretical results versus the Monte Carlo estimators (MCEs) in Table 1, where

Table 1: Infinite Time Horizon P/E

u	n	$\Psi(u)$	$\hat{\Psi}(u)$	σ_{MCE}	S_{MCE}	ε_R
31.9	100	0.05	0.0498	4.5×10^{-4}	5.00×10^{-4}	4.0×10^{-3}
31.9	1000	0.05	0.0499	1.4×10^{-4}	1.41×10^{-4}	2.0×10^{-3}
16.7	1000	0.20	0.1997	5.7×10^{-4}	5.90×10^{-4}	1.5×10^{-3}

$\sigma_{MCE} := \left(\sigma_\theta^2/n\right)^{1/2}$ is the standard error of the MCE, S_{MCE} is the corresponding estimator, and the relative error is

$$\varepsilon_R := \mid 1 - \hat{\Psi}(u)/\Psi(u) \mid .$$

Notice the good fitness between the standard error σ_{MCE} and its estimator S_{MCE}. Obviously, we have better aproximations to $\Psi(u)$ taking larger samples because the MCEs are consistent.

4.2 Example P/U

Let us assume that the claims size distribution is uniform over $(0, 1)$. First of all, we need to find the distibution F_θ, and then we have to show an expression for the conjugate process $R^T_{\theta_1(1-\Delta)}$. From (1)

$$F_\theta(x) = \frac{e^{(\theta-\theta_0)x} - 1}{e^{\theta-\theta_0} - 1}, \quad 0 < x < 1.$$

Recall that under P_θ, Z_t is also a compound Poisson process with parameter

$$\lambda_\theta = \left(e^{\gamma+\theta_1\Delta} - 1\right)/(\gamma + \theta_1\Delta).$$

From Proposition 2.6 and (8) we obtain that in the finite horizon case, and letting $\gamma_0 := \gamma + \theta_1\Delta$,

$$(11) \quad R^T_{\theta_1(1+\Delta)} = \exp\left(-\gamma_0 Y_\tau + \tau\left(\frac{e^{\gamma+\theta_1} - 1}{\gamma_0}\right) - 1 - \gamma_0 c\right) \cdot I\{\tau < T\},$$

whereas in the infinite time horizon

$$R_{\theta_1(1+\Delta)} = \exp\left(-\gamma_0 Y_\tau + \tau\left(\frac{e^{\gamma+\theta_1}-1}{\gamma_0}\right) - 1 - \gamma_0 c\right). \tag{12}$$

Let $\gamma = 0.05$. Then from Lundberg's equation

$$\frac{e^{\gamma}-1}{\gamma} - 1 - c\gamma = 0,$$

we get $c = 0.508439$. Moreover, simple computations show that $-\theta_0 = 0.025078$ and $\theta_1 = \gamma + \theta_0 = 0.024922$.

Unfortunately, there are no theoretical results for the P/U case, so we cannot compare the real and the estimated values like we did under the P/E assumptions. However, it is possible to estimate the variance of the conjugate process and to compute the estimator S_{MCE} of the standard error; see Table 2.

Table 2: Infinite Time Horizon P/U

Δ	$\hat{\Psi}(u)$	$\hat{\sigma}^2_\theta$	S_{MCE}
1.00	0.199	1.5×10^{-2}	1.2×10^{-2}
0.10	0.223	6.2×10^{-4}	2.5×10^{-3}
0.05	0.220	9.9×10^{-5}	9.9×10^{-4}
0.00	0.220	4.2×10^{-6}	2.0×10^{-4}

The computations were made with $n = 100$, and $u = 30$.

Observe that the best estimation occurs when $\Delta = 0$, which is consistent with the asymptotic optimality proved by Asmussen [2].

5 Concluding remarks

In the previous sections we have introduced MCEs for the ruin probability using conjugate processes. In particular, we have shown formulations for the conjugate process under the P/U assumptions for both finite (11) and infinite (12) time horizons.

The P/U case has not been discussed enough in the literature, and so it is suitable for the simulation approach.

Finally it is important to mention two main advantages of the MCEs using conjugate processes: (i) the relative simplicity of the formulation, and (ii) the minimum computational resources needed compared with the diffusion approach [1,2], and the martingale approach [6].

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